

Mechanical Efficiency of a Stirling Engine

Oliver Burch

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1 Abstract

This experiment was designed to use a model Stirling engine to measure changes in temperature and frequency to be able to calculate mechanical efficiency of the system. Using this the result $\eta = 13 \pm 1.7 \%$ was produced where η is mechanical efficiency. This is very similar to the numerical result calculated by Quinn. M and Yeadon. W in their published report [1] on a very similar experiment. This has a small percentage difference between experiments of 3%. A critical limitation in our model was the lack of data produced. This meant data had to be extrapolated which produced some systematic error in results. This is shown and accounted for in the Discussion section.

2 Introduction

Efficiency is an important measure of how effective a system is at converting all input energy to output energy. Thermodynamic efficiency "refers only to processes in which heat is transformed into work" [2]. Measuring mechanical efficiency of a thermodynamic system presents many challenges as consequence of environmental factors; namely, but not limited to, temperature changes in the room or of the object associated with the measurements. Measuring efficiency has numerous uses in the real world. Some include car engines [3] or efficiency of power plants [4]. "A better understanding of engine efficiency leads to less waste of fossil fuels and therefore less environmental damage" [5].

One such method of measuring mechanical efficiency is to use a Stirling engine. A cup of boiling water drives the engine and frequency of rotations is recorded using a light-gate and oscilloscope. Using this method, Quinn. M and Yeadon. W recorded thermal efficiencies of $10 \pm 2\%$ [1]. This is similar to the method we used in the lab to record results as is demonstrated in the Methods section. A second method is to set up a cold and hot reservoir and measure temperatures at each to calculate work done by the hot reservoir to the cold. This was an experiment conducted by Queens Mary University London [6], however no recorded readings are given for this experiment. The difference between this experiments is the measurements taken rely on frequency (Quinn. M) and work being done (Queen Mary). While these experiments work, they provide issues. Quinn's method provides a small range of temperatures to take data from, while Queen Mary's experiment revolves around interpolated data to measure differences in temperature.

In this investigation we aim to provide a further analysis of thermal efficiency in a Stirling engine. We aim to perform a similar experiment to that of Yeadon. W and Quinn. M while expanding on their research to provide a larger database; proving that frequency is directly proportional to the square root of temperature difference. This will allow us to perform calculations of efficiency from the slope of our graph. To do this we used dry ice on the top plate of the engine to provide a much larger temperature difference and found

that our expectations were met ¹. It's clear that these changes may have made a suitable contribution as is seen in the Results and Discussions sections by the fact that a smaller error and larger efficiency than that of Quinn and Yeadon's experiment are recorded.

3 Theory

A Stirling engine functions using a predicted cycle assuming an ideal gas: 1) Isochoric heating, 2) Isothermal expansion, 3) Isochoric heating, 4) Isothermal contraction. We can use this to work out thermodynamic efficiency as each stage of the cycle behaves in a specific way. This allows us to produce associated equations for each stage, finally rearranging these into an equation for mechanical efficiency, ' η ', of the engine. This is the ratio of useful work done as the wheel spins by the work done by the gas:

$$\eta = \frac{W_{wheel}}{W_{net}} \quad (1)$$

Work done on the wheel can be calculated assuming energy is kinetic:

$$W_{wheel} = \frac{1}{4}Mv^2 \quad (2)$$

Work done by the gas can be calculated using the ideal gas equation and as stages 1 and 3 are opposite, overall work done between them is 0J. Therefore only work done in stages 2 and 4 need to be accounted for. Hence net work done, W_{net} , can be calculated by:

$$W_{net} = W_{in} - W_{out} = nR(T_H - T_C)\ln(CR) \quad (3)$$

So mechanical efficiency can be calculated using:

$$\eta = \frac{Mv^2}{4nR(T_H - T_C)\ln(CR)} \quad (4)$$

This shows that theoretically mechanical efficiency is directly proportional to velocity squared and inversely proportional to change in temperature. *Figure 2* will later show this in the Analysis section. A few assumptions for this theory to work were needed. The wheel had to be modelled as a disk to calculate W_{wheel} , there was no friction accounted for between the engine and wheel, the model was assumed to be ideal. It was also assumed that varying temperature would vary velocity.

¹Dry ice was used as opposed to any other substance as it is very cold (approximately -79 degrees Celsius) and is cheap to import in bulk.

4 Methods

For this experiment, a system as shown in Figure 1 was set up. Dry ice and boiling water were used to create a large temperature gap between the top and bottom plates of the Stirling Engine to acquire the range of temperatures shown in *Figure 2*. These were recorded via the thermocouples attached which fed data to the thermometer. At the same time, the temperature difference caused the piston to oscillate, turning the wheel. This data was measured by a light gate which fed data to an oscilloscope (model GW INSTEK GDS-1052-U). This was then analysed to measure frequency at different temperatures. This data could then be put into a manipulated *equation 4* as shown in the Analysis section (*equation 7*).

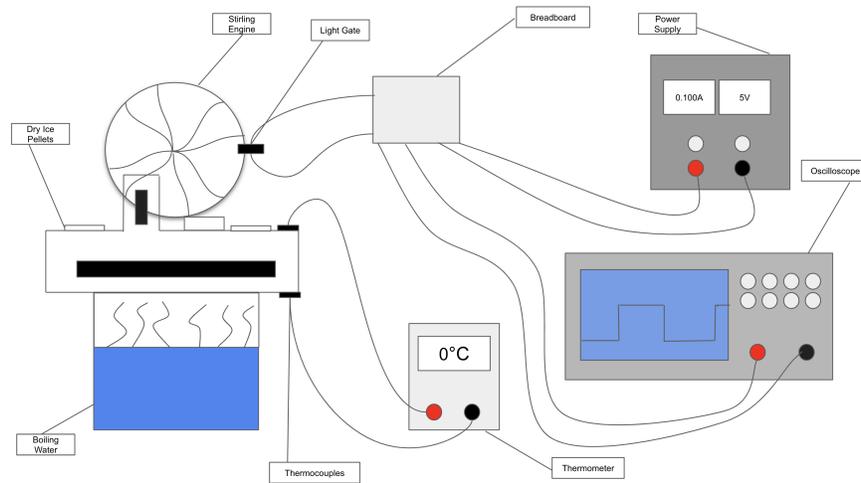


Figure 1: The setup used for this experiment. Thermocouples feed data to a thermometer to measure temperature differences, while a light gate feeds data to an oscilloscope to measure frequency.

Error in this experiment results from small differences in reading frequency from the oscilloscope. It was difficult to tell exactly where to take readings from on the screen due to background noise, such as friction of the wheel, having an effect on the quality of the plot produced. The thermometer was also only correct to whole integers meaning temperatures were rounded. This leaves an error of ± 1 degree Celsius. The repercussions on the final recorded result of Mechanical efficiency are discussed further in the Discussion section.

5 Analysis & Results

Our investigation measured one set of initial results of frequency and temperature difference, and a second set of repeats so that we could take a weighted mean of both as shown in *Figure 2*. Our data presented a strong positive correlation in both initial results and repeats, and showed no outliers. Error bars represent error in temperature and frequency. These remained constant at $\pm 1^{\circ}\text{C}$ and $\pm 4\text{Hz}$.

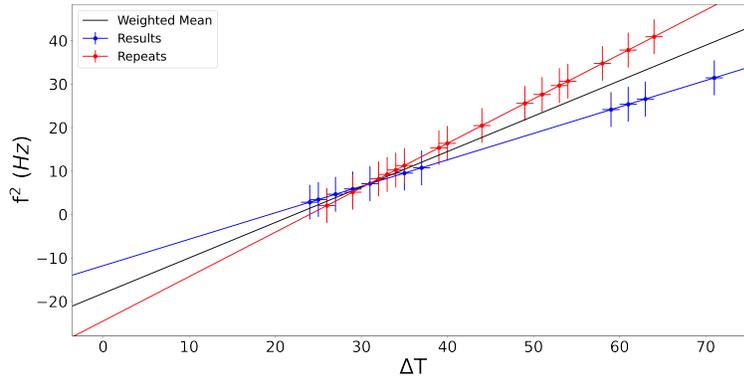


Figure 2: Measurements of a Stirling engine. A least squares fit of data is shown alongside a weighted fit. The investigation ran twice using dry ice to cool the top plate, and boiling water to heat the bottom. Error bars in the y-data are calculated from a fixed error in calculations of frequency from an oscilloscope. Error bars in x-data are calculated from the resolution of the two thermometers used.

Analysis of this data was performed to allow calculations of mechanical efficiency to be performed. *Figure 2* is a visual representation of the equation:

$$f^2 = m\delta T + c \quad (5)$$

where m is the slope:

$$m = \frac{\eta 4nR \ln(CR)}{r^2 4\pi^2 M} \quad (6)$$

No linearisation of this data was required to present the slope in a workable way. A weighted fit is plotted in *Figure 2* to calculate a more accurate slope of the mean of the plotted fits. Error bars are included in analysis of data, however as error in y-data and x-data is constant, no work on the error needs to be done.

The results of this regression are recorded for the weighted mean and are as follows. The value of gradient is $m = 0.8 \pm 0.1$ and intercept was calculated at $-20 \pm 4 \text{ Hz}^2$. As this was a weighted mean, there is no correlation coefficient

as there are no data points, however for results this was $r^2 = 0.84099$ and for repeats this was $r^2 = 0.97529$. Uncertainty in m is constant so remains at 0.8 ± 0.1 . Our investigation set out to measure mechanical efficiency and so To do this we can rearrange *equation 6* to produce:

$$\eta = \frac{r^2 m 4 \pi^2 M}{4nR \ln(CR)} \quad (7)$$

The result therefore comes out to be $\eta = 13 \pm 1.7 \%$.

6 Discussion & Conclusion

In this investigation, we set out to measure a value of mechanical efficiency. A greater slope suggests a greater mechanical efficiency as $m \propto \eta$. This produces a result very similar to predicted values. Slight variance may be caused by a slightly large statistical error of 13%. Systematic error is evident in this experiment from a non-zero intercept. Analysis of data shows that both our values for results and repeats had strong positive correlation, with that of repeats being stronger. Strong values of correlation coefficient suggest both initial results and repeats had high levels of precision. Variance between results and repeats suggests that analysis was limited by only taking two sets of data. Other errors could easily be produced from faulty wiring or parallax error when taking readings from the oscilloscope.

The main finding of this investigation was that $\eta = 13 \pm 1.7 \%$. This result is valid as this is very alike measurements recorded in the similar experiment conducted by Quinn. M and Yeadon. W [1] which produced a result of $\eta = 10 \pm 2 \%$. In future repeats of this experiment we could record longer periods of data to provide more data points, or reduce/increase the temperature by large values to increase the scope of our investigation.

In conclusion, we have measured the mechanical efficiency value of a Stirling engine. The result $\eta = 13 \pm 1.7 \%$ was very similar to predicted results. To improve this experiment further work would be needed to acquire more data plots with a larger temperature difference. This would also produce results with far less systematic error.

References

- [1] Yeadon W, Quinn M. Thermodynamics education for energy transformation: a Stirling Engine experiment. *Physics Education*. 2021 aug;56(5):055033. Available from: <https://dx.doi.org/10.1088/1361-6552/ac142c>.
- [2] Arfken GB, Griffing DF, Kelly DC, Priest J. chapter 24 - THE SECOND LAW OF THERMODYNAMICS. 1984:444-58. Available from: <https://www.sciencedirect.com/science/article/pii/B9780120598601500291>.
- [3] Orfila O, Saint Pierre G, Messias M. An android based ecodriving assistance system to improve safety and efficiency of internal combustion engine passenger cars. 2015;58:772-82.
- [4] Li A, Zhang A, Huang H, Yao X. Measuring unified efficiency of fossil fuel power plants across provinces in China: An analysis based on non-radial directional distance functions. *Energy (Oxford)*. 2018;152:549-61.
- [5] Armaroli N, Balzani V. The Legacy of Fossil Fuels. *Chemistry, an Asian journal*. 2011;6(3):768-84.
- [6] Laboratory Exercise 12 - Thermal Efficiency. Queen Mary University London;. Available from: <https://ph.qmul.ac.uk/sites/default/files/courses/PHY4103/labex12n.pdf>.