A Nod to the Geometry of Holes

Or, How I Failed to Settle a Debate Amongst my Fellow Students

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Abstract

A geometric theory of cavities in a closed bounded host $H \subset \mathbb{R}^3$ is developed via the cavity complex $\overline{\mathrm{conv}(H) \setminus H}$ and a single integer invariant - aperture cardinality α - admitting two equivalent formulations. A duality theorem identifies the static count of aperture components with the kinematic count of access-path homotopy classes. The naive trichotomy ($\alpha = 0$ sealed void, $\alpha = 1$ pocket, $\alpha \geq 2$ through-hole) is shown to be a coarsening of the actual structure: pockets and through-holes are connected by aperture-merge deformations, and every pocket arises as the Hausdorff limit of through-holes. The natural classification is therefore a dichotomy - void versus hole - with the hole class filtered by α as a stratification rather than partitioned as disjoint species. Pockets occupy the lowest stratum: holes whose two ways in have merged into one.

1 Setup

A *host body* is a non-empty, closed, bounded subset $H \subset \mathbb{R}^3$ that is regular such that $H = \overline{\mathrm{int}(H)}$. We write ∂H for its boundary, $\mathrm{int}(H)$ for its interior, and $\mathrm{ext}(H)$ for the unbounded component of $\mathbb{R}^3 \setminus H$.

2 The cavity complex and apertures

The cavity complex of H is

$$\Gamma(H) = \overline{\mathrm{conv}(H) \setminus H}, \tag{1}$$

and a cavity of H is a connected component of $\Gamma(H)$, denoted $C_1, C_2, \ldots, C_n$ where $n = n(H) \in \mathbb{Z}_{\geq 0}$.

For a cavity C_i, its aperture set is

$$A_i = \partial C_i \cap \partial(\mathrm{conv}(H)). \tag{2}$$

The *aperture cardinality* of C_i, written $\alpha(C_i)$, is the number of connected components of A_i.

A cavity C_i is, by definition,

- a *sealed void* if $\alpha(C_i) = 0$;
- a *pocket* if $\alpha(C_i) = 1$;
- a *through-hole* if $\alpha(C_i) \geq 2$.

Every cavity is exactly one of these, since the three conditions partition $\mathbb{Z}_{\geq 0}$ and $\alpha(C_i) \in \mathbb{Z}_{\geq 0}$ for every cavity. The trichotomy is convenient but not final: §7 shows that pockets and through-holes are not isolated species but adjacent strata of a single category, joined by aperture-merge transitions. We retain the present language for now and revisit the classification once the deformation theory of §6 is in place.

Examples

- If H is a solid coffee mug, $\Gamma(H)$ has two cavities: the cup-cavity ($\alpha = 1$) and the handle-hole ($\alpha = 2$).
- If H is a solid ball, $\Gamma(H) = \emptyset$ and H has no cavities.
- If H is a pair of trousers, each pocket is a cavity with $\alpha = 1$, and each trouser leg is a through-hole with $\alpha = 2$.

3 The kinematic formulation

The kinematic formulation reframes this data in terms of access. The *admissible path space* of H is

$$\mathcal{P}(H) = \left\{ \gamma : [0,1] \to \overline{\mathrm{conv}(H)} \setminus \mathrm{int}(H) \;\middle|\; \gamma \text{ continuous} \right\}. \tag{3}$$

An *access path* into C_i is a path $\gamma \in \mathcal{P}(H)$ satisfying

- $\gamma(0) \in \partial(\mathrm{conv}(H))$;
- $\gamma(1) \in \overline{C_i}$;
- $\gamma((0,1)) \subseteq \overline{C_i}$,

the last of which implies the path does not enter through another cavity, since $\overline{C_i} \cap \partial(\mathrm{conv}(H)) = A_i$. The set of access paths into C_i is denoted $\mathcal{A}_i$.

Two access paths $\gamma_0, \gamma_1 \in \mathcal{A}_i$ are *access-homotopic*, written $\gamma_0 \simeq \gamma_1$, if there exists a continuous map

$$h : [0,1]^2 \to \overline{\mathrm{conv}(H)} \setminus \mathrm{int}(H) \tag{4}$$

such that $h(0,t) = \gamma_0(t)$, $h(1,t) = \gamma_1(t)$, $h(s,0) \in A_i$, and $h(s,t) \in \overline{C_i}$ for all $s \in [0,1]$ and $t \in (0,1]$.

The *kinematic aperture cardinality* of C_i is

$$\alpha_{\mathrm{kin}}(C_i) = |\mathcal{A}_i / {\simeq}|, \tag{5}$$

with the convention $\alpha_{\mathrm{kin}}(C_i) = 0$ when $\mathcal{A}_i = \emptyset$.

4 The duality theorem

Theorem 1 (Static–kinematic duality)**.** *For any host body H and any cavity C_i of H,*

$$\alpha(C_i) = \alpha_{\mathrm{kin}}(C_i). \tag{6}$$

Proof. We require three claims.

Claim 1. *If $A_i = \emptyset$, then $\mathcal{A}_i = \emptyset$.* Suppose $A_i = \emptyset$. Then $\partial C_i \cap \partial(\mathrm{conv}(H)) = \emptyset$ and $\mathrm{int}(C_i) \subseteq \mathrm{int}(\mathrm{conv}(H))$, so $\overline{C_i} \cap \partial(\mathrm{conv}(H)) = \emptyset$. Any $\gamma \in \mathcal{A}_i$ satisfies $\gamma(0) \in \partial(\mathrm{conv}(H))$ and (by the third access-path condition extended by continuity to $t = 0$) $\gamma(0) \in \overline{C_i}$, contradiction.

Claim 2. *If $\gamma_0, \gamma_1 \in \mathcal{A}_i$ satisfy $\gamma_0(0), \gamma_1(0) \in K$ for the same connected component K of A_i, then $\gamma_0 \simeq \gamma_1$.* Pick a path $\beta : [0, 1] \to K$ joining $\gamma_0(0)$ to $\gamma_1(0)$ and a basepoint $p_0 \in \mathrm{int}(C_i)$. For each $s \in [0, 1]$, let η_s be a path in $\overline{C_i}$ from $\beta(s)$ to p_0 (exists since $\overline{C_i}$ is path-connected). Concatenating η_s with a chosen path from p_0 to a point on the line segment between $\gamma_0(1)$ and $\gamma_1(1)$, parameterised by s, yields a continuous family of access paths interpolating γ_0 and γ_1, with $h(s, 0) = \beta(s) \in K \subseteq A_i$ and $h(s, t) \in \overline{C_i}$ for $t \in (0, 1]$.

Claim 3. *If $\gamma_0(0) \in K_0$ and $\gamma_1(0) \in K_1$ for distinct components K_0, K_1 of A_i, then $\gamma_0 \not\simeq \gamma_1$.* Suppose h is an access homotopy. The map $s \mapsto h(s, 0)$ is continuous from $[0, 1]$ to A_i, beginning in K_0 and ending in K_1. Continuity then forces K_0 and K_1 to lie in the same component of A_i, contradiction.

The three claims establish a bijection between connected components of A_i and equivalence classes of $\mathcal{A}_i$ under $\simeq$, proving $\alpha(C_i) = \alpha_{\mathrm{kin}}(C_i)$. □

The duality theorem (6) states that aperture cardinality is intrinsically meaningful: it can be defined geometrically or kinematically, and the two counts agree. This distinguishes a natural definition from a conventional one.

A kinematic restatement of classification: a cavity C_i is

- a *sealed void* if there is no access path into C_i;
- a *pocket* if every access path is access-homotopic to every other;
- a *through-hole* if there exist mutually non-homotopic access paths.

Therefore a pocket is a cavity in which every entry path is, up to homotopy, the same path. *The way in is the way out.*

5 Geometric invariants

Beyond classification, the framework supports geometric invariants. The *depth* of a cavity C_i with $\alpha(C_i) \geq 1$ is

$$d(C_i) = \sup_{q \in C_i} \inf_{\substack{\gamma \in \mathcal{A}_i \\ \gamma(1) = q}} \mathrm{length}(\gamma); \tag{7}$$

that is, the supremum over interior points of the minimum access-path length needed to reach that point.

The *aperture area* of C_i is the 2-dimensional Hausdorff measure

$$\mathrm{ar}(C_i) = \mathcal{H}^2(A_i). \tag{8}$$

For a pocket C_i, the *aspect ratio* is

$$\rho(C_i) = \frac{d(C_i)^2}{\mathrm{ar}(C_i)}. \tag{9}$$

These invariants distinguish pockets quantitatively: a trousers pocket is shallow with a wide aperture (low ρ); a cave is deep with a narrow aperture (high ρ). The aspect ratio captures the qualitative difference between 'wide-shallow' and 'deep-narrow' containment.

6 Smooth deformations and robustness

A *smooth deformation* of H is a continuous one-parameter family $\{H_t\}_{t\in[0,1]}$ of host bodies with $H_0 = H$, varying continuously in the Hausdorff metric.

Let $\{H_t\}$ be a smooth deformation. The aperture cardinality $\alpha(C_i^{(t)})$ of each cavity is locally constant in t except at a discrete set of critical values where the homotopy type of $\Gamma(H_t)$ changes. The cavity complex $\Gamma(H_t)$ varies continuously in the Hausdorff metric. Connected components of $A_i^{(t)}$ are stable under continuous deformations except at moments when components merge or split - events that can be characterised as critical values of an associated Morse-theoretic functional on the deformation parameter. Such moments form a discrete set under generic conditions.

Remark. Pockethood is fragile but locally stable. A pocket survives small perturbations but flips classification at critical events such as piercing or sealing. The two non-trivial critical events - the merging of two aperture components into one and the splitting of one into two - are not auxiliary technicalities but the central transitions of the theory. §7 names them and shows they are the deformations that connect pockets to through-holes.

7 Pockets as limit holes

Originally cavities were partitioned into three classes by aperture cardinality: sealed voids ($\alpha = 0$), pockets ($\alpha = 1$), and through-holes ($\alpha \geq 2$). The trichotomy is accurate but coarse. The two non-degenerate classes - pockets and through-holes - are not isolated species. They are connected by a continuous transition: a pocket is what a through-hole becomes when its apertures merge, and a through-hole is what a pocket becomes when its aperture is bisected. This section will make this precise and argue that the natural taxonomy is therefore a dichotomy: *voids* and *holes*.

7.1 Holes

A cavity C_i is a *hole* if $\alpha(C_i) \geq 1$. The cavities of H partition cleanly into voids ($\alpha = 0$) and holes ($\alpha \geq 1$). Within holes, a refinement by aperture cardinality survives.

The redefinition is shelved until we show the strata are geometrically connected. It then follows that this is our next step.

7.2 Aperture-merge deformations

We work in the smooth-deformation setting. Fix a host body H_0 with a distinguished cavity $C^{(0)}$.

A smooth deformation $\{H_t\}_{t\in[0,1]}$ with distinguished cavities $\{C^{(t)}\}$ is an *aperture-merge deformation* of $C^{(0)}$ if

- $\alpha(C^{(t)}) = 2$ for every $t \in (0, 1]$, and
- $\alpha(C^{(0)}) = 1$,

and the two components $A_1^{(t)}, A_2^{(t)}$ of the aperture set converge in Hausdorff distance to the single component $A^{(0)}$ as $t \to 0^+$. Symmetrically, a deformation is an *aperture-split deformation* of a pocket $C^{(0)}$ if it is the time-reverse of an aperture-merge deformation, taking α from 1 at $t = 0$ to 2 for $t > 0$.

As the deformations exist, we must show every pocket admits one.

Theorem 2 (Pocket-hole unification)**.** *Let C_0 be a pocket of H_0. There exists an aperture-split deformation $\{H_t\}_{t\in[0,1]}$ with $H_t|_{t=0} = H_0$ and $C^{(t)}$ a through-hole of H_t for every $t \in (0, 1]$. Equivalently, C_0 arises as the Hausdorff limit, as $t \to 0^+$, of through-holes $C^{(t)}$.*

Proof. Let $A_0 \subset \partial(\mathrm{conv}(H_0))$ be the single aperture component of C_0. Since A_0 is a non-empty connected open subset (in the relative topology of $\partial(\mathrm{conv}(H_0))$) of a topological 2-manifold, choose a smooth, simple arc $\gamma : [0, 1] \to \overline{A_0}$ with $\gamma(0), \gamma(1) \in \partial A_0$ and $\gamma((0, 1)) \subseteq \mathrm{int}(A_0)$. Such an arc exists provided A_0 is not a single point; the measure-zero pinhole case is treated in the remark below.

For $t \in [0, 1]$, let $W_t \subset \mathbb{R}^3$ denote the closed t-neighbourhood of γ in the ambient metric, intersected with the closed half-space on the cavity side of $\partial(\mathrm{conv}(H_0))$ (so that W_t is a thin wedge protruding into C_0 along γ). Define

$$H_t = H_0 \cup W_t.$$

The family is well-defined.

Claim 1. H_t is a host body for every $t \in [0, 1]$. The union of two regular closed bounded sets in $\mathbb{R}^3$ is closed and bounded; regularity ($H_t = \overline{\mathrm{int}(H_t)}$) follows because W_t is regular and meets H_0 along a set with non-empty interior in H_0 for $t > 0$.

Claim 2. $C^{(t)}$ has aperture cardinality 2 for every $t \in (0, 1]$ small enough. The cavity complex $\Gamma(H_t)$ is the closure of $\mathrm{conv}(H_t) \setminus H_t$. For t small, $\mathrm{conv}(H_t) = \mathrm{conv}(H_0)$ (since $W_t \subset \mathrm{conv}(H_0)$), so $\Gamma(H_t) = \overline{\Gamma(H_0) \setminus W_t}$. Restriction to $\partial(\mathrm{conv}(H_0))$ gives

$$A^{(t)} = A_0 \setminus N_t(\gamma),$$

where $N_t(\gamma)$ is the open t-neighbourhood of γ in $\partial(\mathrm{conv}(H_0))$. By construction, γ joins two boundary points of A_0 through the relative interior, so $A_0 \setminus N_t(\gamma)$ has exactly two connected components for all $t > 0$ small enough. Hence $\alpha(C^{(t)}) = 2$.

Claim 3. $H_t \to H_0$ in Hausdorff distance as $t \to 0^+$. The wedge W_t has Hausdorff distance to γ bounded by t, and $\gamma \subset \partial H_0$, so $d_H(H_t, H_0) \to 0$.

Claim 4. The apertures $A_1^{(t)}, A_2^{(t)}$ of $C^{(t)}$ converge to A_0. Each $A_i^{(t)}$ is contained in A_0, and their union $A_1^{(t)} \cup A_2^{(t)} = A_0 \setminus N_t(\gamma)$ has Hausdorff distance to A_0 at most t.

The four claims together establish the theorem. □

Remark (pinhole apertures). If A_0 is a single point or a one-dimensional arc, the construction degenerates - there is no chord γ in the relative interior. The unification still holds, but via a different construction: perturb H_0 first to a host body whose pocket has a 2-dimensional aperture (which is generic in the smooth-deformation sense), then apply the theorem. The non-generic pinhole pockets are limits of generic pockets, which are limits of through-holes. Transitivity of Hausdorff limits closes the gap.

Remark (the converse). The construction is reversible. Given a through-hole with apertures A_1, A_2 that lie on a common connected component of $\partial(\mathrm{conv}(H)) \cap \partial H$, one can shrink the strip of host material separating them; at the moment the strip degenerates, the apertures merge and the cavity becomes a pocket. The merge direction is the more natural philosophically: holes *lose* apertures by merging.

7.3 Kinematic merging

The unification has a transparent kinematic restatement. $\alpha_{\mathrm{kin}}(C_i)$ counts homotopy classes of access paths into C_i. By the duality theorem we established, $\alpha_{\mathrm{kin}} = \alpha$.

Proposition 3. *Let $\{H_t\}$ be an aperture-merge deformation with distinguished cavities $\{C^{(t)}\}$, taking $\alpha = 2$ at $t > 0$ to $\alpha = 1$ at $t = 0$. For each $t > 0$, fix representatives $\gamma_1^{(t)}, \gamma_2^{(t)}$ of the two access-homotopy classes. Then there exist limits $\gamma_1, \gamma_2 \in \mathcal{A}_0$ of $\gamma_1^{(t)}, \gamma_2^{(t)}$ as $t \to 0^+$ such that $\gamma_1 \simeq \gamma_2$ in $\mathcal{A}_0$.*

Proof. The path spaces $\mathcal{P}(H_t)$ form a continuous family; access paths are compact in the supremum topology. Compactness gives subsequential limits $\gamma_1, \gamma_2 \in \mathcal{A}_0$. Both terminate in $\overline{C_0}$ and originate at points of A_0 (the limit of $A_1^{(t)}$ and of $A_2^{(t)}$, which coincide). Since $\alpha(C_0) = 1$ and the duality theorem identifies homotopy classes with aperture components, the limit paths lie in the unique homotopy class, hence $\gamma_1 \simeq \gamma_2$. □

The two homotopy classes of through-hole access paths collapse, in the limit, to one. Where the through-hole had two ways in, the pocket has one - because the two have become indistinguishable. Said the other way round: *the pocket has one homotopy class because two have merged.* This is the kinematic counterpart of aperture-merging, and it is what the slogan "the way in is the way out" captures from the inside of the limit.

7.4 Stratification of host-body space

Let $\mathcal{H}_*$ denote the space of host bodies with a single distinguished hole, equipped with the Hausdorff metric. Stratify $\mathcal{H}_*$ by aperture cardinality:

$$\mathcal{H}_*^{(k)} = \{H \in \mathcal{H}_* : \alpha(C(H)) = k\}, \qquad k \geq 1.$$

Theorem 4 (Stratification)**.** *For every $k \geq 1$, the closure $\overline{\mathcal{H}_*^{(k+1)}}$ contains $\mathcal{H}_*^{(k)}$. In particular, $\mathcal{H}_*^{(1)} \subseteq \overline{\mathcal{H}_*^{(2)}}$: every pocket is a Hausdorff limit of through-holes.*

Proof. The case $k = 1$ is Theorem 2. For $k \geq 2$, an analogous wedge construction performed on a chord that lies entirely within *one* of the k components of A_0 produces an aperture-split deformation taking that component to two, raising aperture cardinality from k to $k+1$. Reversing the deformation gives an aperture-merge from $k+1$ to k, exhibiting the inclusion. □

The picture is that $\mathcal{H}_*$ is filtered:

$$\mathcal{H}_*^{(1)} \subset \overline{\mathcal{H}_*^{(2)}} \subset \overline{\mathcal{H}_*^{(3)}} \subset \cdots$$

Pockets sit at the bottom of the filtration - they are the most degenerate holes. Through-holes are generic in a precise sense: $\mathcal{H}_*^{(2)}$ has non-empty interior in $\mathcal{H}_*$, while $\mathcal{H}_*^{(1)}$ does not. Pockethood is a positive, codimension condition. This refines the remark that pockethood is fragile; infact not merely fragile, but *boundary.*

7.5 Reframing the trichotomy

Theorem 2 licenses a substantive reframing. The original classification

$$\{\text{sealed void}\} \sqcup \{\text{pocket}\} \sqcup \{\text{through-hole}\}$$

is replaceable, without loss of information, by

$$\{\text{void}\} \sqcup \{\text{hole}\}, \qquad \text{hole stratified by } \alpha \geq 1.$$

The first partition treats the three classes as disjoint species. The second recognises that two of them - pocket and through-hole - are the same species at different points of a continuum, indexed by an integer that admits continuous transitions through aperture-merge and aperture-split. Pockethood is then not a kind of cavity but a *stratum* within hole-space: the codimension-one boundary on which the apertures of higher strata coalesce.

Aphoristically: *a pocket is a hole whose ways in have been identified.* The way in is the way out because the two ways in - which would have made it a through-hole - have become a single way. Pockets are not adjacent to holes; they are limits of holes. The trichotomy is an artefact of static observation. Permit the host body to deform, and the trichotomy collapses.

8 Content functions and occupancy

A *content function* on a cavity C_i is a measurable function $f : C_i \to V$ for some target space V representing physical states (matter density, EM field, quantum state, ...). The content $f \equiv 0_V$ is the *empty content*.

Pockethood is a property of C_i; occupancy is a property of f. They are formally independent. This emerges from the proof that the classification of C_i depends only on $\alpha(C_i)$, a function of H and C_i alone. The content function f is defined on C_i but does not enter the classification. Hence the structures are independent.

A few remarks:

- A cavity C_i is a hole regardless of whether the target space V admits any non-zero state at the scale of C_i. Geometric regions of pocket type below the threshold of physical matter remain pockets in the present framework.
- Empty content is treated as a particular section of the content bundle, not as the absence of a section. This formalises the Buddhist '*śūnyātā*' observation: a pocket containing nothing still has a content function, namely the zero function, and that function is no less a function than any other. Empty pockets are not unfurnished; they are furnished with the zero element of V.

9 Manifold extensions

The framework extends to a Riemannian manifold (M, g) as follows. Replace $\mathbb{R}^3$ with M and $\text{conv}(H)$ with the geodesic-convex hull $\text{conv}_g(H)$, defined as the smallest geodesically-convex subset of M containing H.

All definitions, the duality theorem (Theorem 1), and the unification theorem (Theorem 2) generalise, with minor modifications to the proofs to handle path connectivity in the manifold setting.

For pseudo-Riemannian manifolds, additional care is needed: access paths must be specified as spacelike, timelike, or causal, and the resulting aperture cardinalities will generally differ across these.

10 Edge cases and pathologies

- If A_i has fractal boundary, $\alpha(C_i)$ remains well-defined as the number of connected components, even when A_i has fractional Hausdorff dimension. The kinematic formulation is robust to this.
- A cavity may have $A_i \neq \emptyset$ but $\mathcal{H}^2(A_i) = 0$. Such cavities are still pockets since $\alpha(C_i) = 1$. The kinematic interpretation strains here: a measure-zero aperture admits only a measure-zero set of access paths in any reasonable measure on path space. Pinhole pockets receive the unification of §7 only via the perturbation argument noted there.
- If A_i has infinitely many connected components, then $\alpha(C_i) = \infty$. Such cavities remain through-holes and most theorems generalise.
- $H = \overline{\text{int}(H)}$ excludes pathological cases. For non-regular hosts, the framework can be applied to the regularisation $\overline{\text{int}(H)}$, with the understanding that some structure is lost in passing to the regularisation.
- If H is disconnected, the cavity complex and aperture cardinality are still well-defined component-by-component, though the convex hull operation will conjoin disconnected components in ways that may produce spurious cavities. For most applications, restricting to connected hosts is appropriate.

11 Summary

The geometric theory of pockets rests on a single invariant - aperture cardinality - admitting two equivalent formulations:

1. *Statically*, as the number of connected components of the cavity complex's contact with the convex hull boundary.
2. *Kinematically*, as the number of homotopy classes of access paths.

Their equivalence (Theorem 1) is the framework's first foundational result.

The natural classification it supports is not the trichotomy with which the paper began but a dichotomy: cavities are *voids* ($\alpha = 0$) or *holes* ($\alpha \geq 1$). The hole class is further filtered by aperture cardinality, and the filtration is geometrically connected: each stratum lies in the closure of the next (Theorem 4). Pockets occupy the lowest stratum - the codimension-one boundary on which higher strata's apertures coalesce. They are not a separate kind of cavity but the limit configuration of through-holes. This is the framework's second foundational result, and it is what licenses calling pockets holes.

Geometric invariants (depth, aperture area, aspect ratio) refine classification quantitatively. Robustness under deformation is locally generic, with critical events identified as aperture-merge and aperture-split transitions. Occupancy is formally independent of cavity type. The framework extends to manifolds and absorbs edge cases without catastrophic failure.

In this account, a pocket is a hole whose two ways in have become one. *The way in is the way out, because the two ways in have merged.* The slogan and the theorem are the same statement read from opposite sides.

Acknowledgements

This work was conducted with the use of Anthropic's Claude AI. What started as a friendly debate amongst my fellow students at the University of Sheffield about whether all holes were pockets emerged into a much deeper conversation with Claude's Opus model. The framework of §§1–6 was the result of direct prompts asking Claude to consider a cave that shrunk over an infinitely long distance as you walked through it and shrunk with it, and whether a shape with pocket-like geometry which cannot contain any matter is still a pocket. The unification result of §7 came from a later conversation, prompted by asking Claude to show that all pockets are holes by exploring how an $\alpha = 1$ aperture configuration merges into an $\alpha = 2$ one.

Notation

H	Host body
$\partial H, \mathrm{int}(H), \mathrm{ext}(H)$	Boundary, interior, exterior (exterior = $\mathbb{R}^3 \setminus H$)
$\mathrm{conv}(H)$	Convex hull of H (smallest convex set containing the shape)
$\Gamma(H)$	Cavity complex of H (captures all missing material in H)
C_i	i-th cavity of H
A_i	Aperture set of C_i (the 'doorway')
$\alpha(C_i)$	Static aperture cardinality (count of 'doorways')
$\mathcal{P}(H)$	Admissible path space (continuous motion within boundary)
$\mathcal{A}_i$	Access paths into C_i (act of walking through 'doorway')
$\simeq$	Access homotopy (deformation of one path onto another)
$\alpha_{\mathrm{kin}}(C_i)$	Kinematic aperture cardinality (count of distinct ways in)
$d(C_i)$	Depth of C_i
$\mathrm{ar}(C_i)$	Aperture area of C_i
$\rho(C_i)$	Aspect ratio of C_i
f	Content function on a cavity
$\{H_t\}$	One-parameter family of host bodies (smooth deformation)
W_t	Wedge used in the aperture-split construction (§7)
$\mathcal{H}_*^{(k)}$	Stratum of host-body space with $\alpha = k$